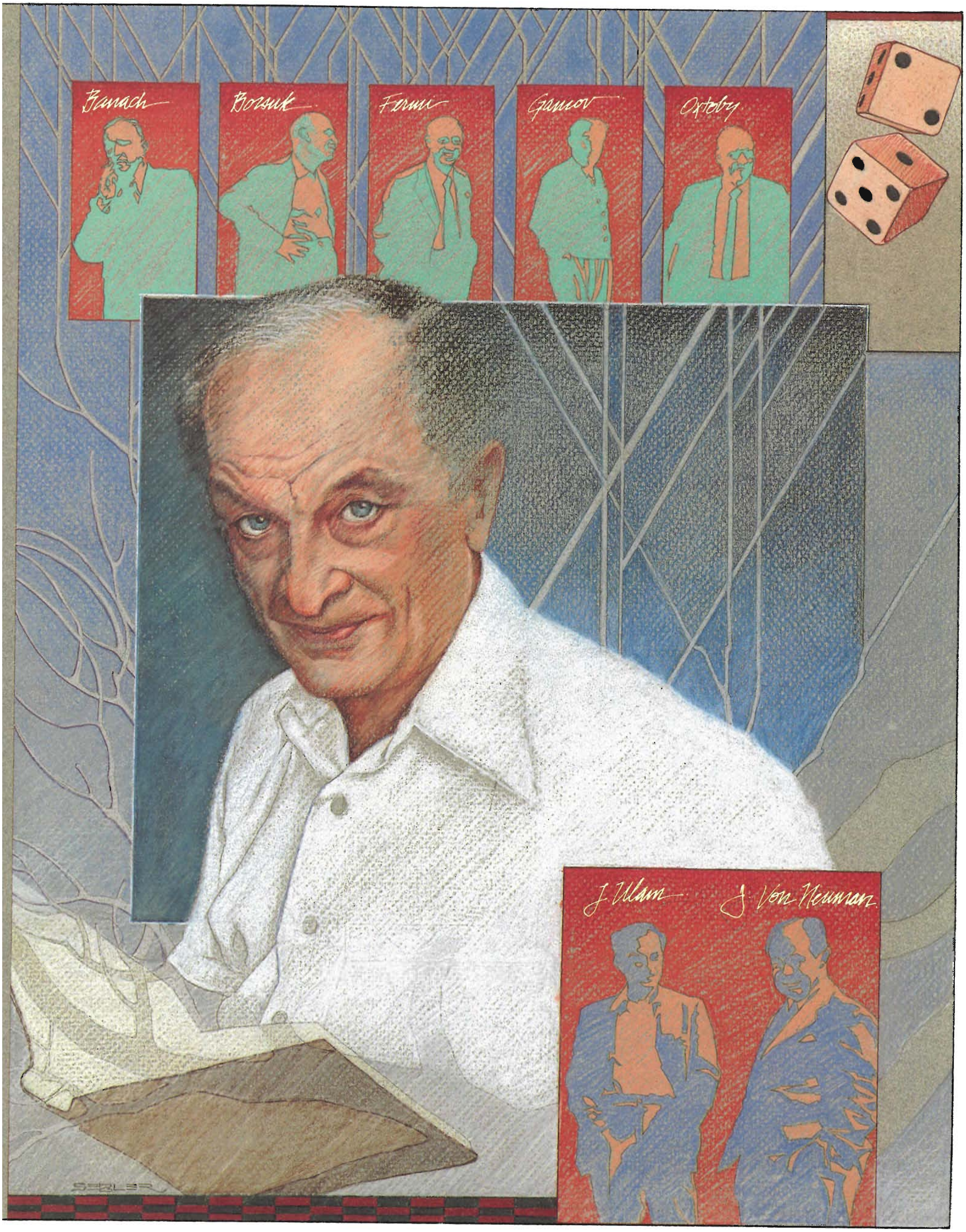
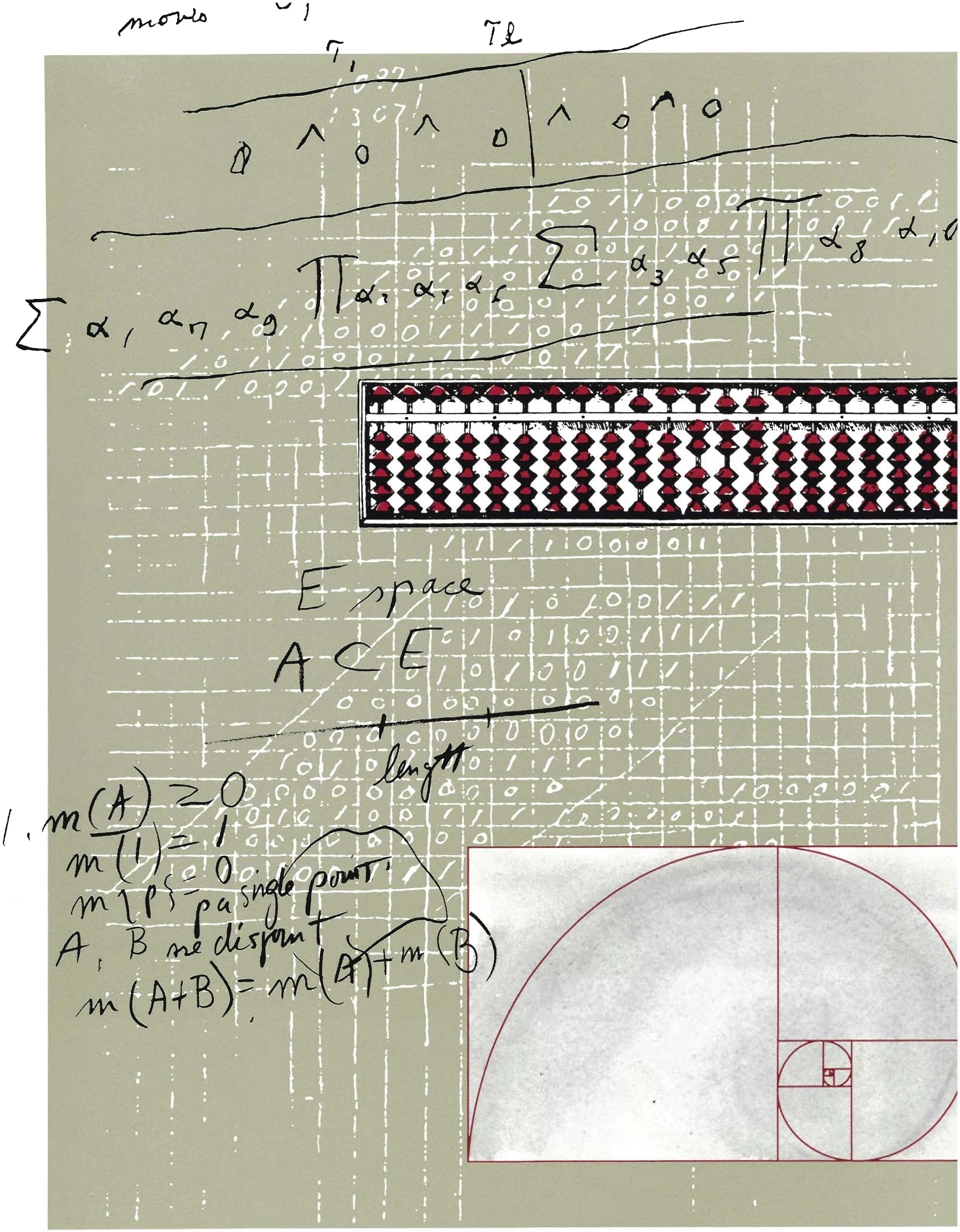
PART II

The Ulam Legacy

TO INTERDISCIPLINARY SCIENCE

"There is not a single mathematician who in the slightest degree reminds one of Stan . . . Mathematicians are much more in a mold, because finally the only method they have is logic. There is imagination and all that, but there are no experiments, no external things like those that tickle a physicist's imagination. Most mathematicians don't experiment at all. Stan did. There is nobody like him, absolutely nobody." (Mark Kac, 1984) The contents that follow were inspired by that instinct to experiment, not only in mathematics, but in physics and biology as well.


Banach
Borsuk
Fermi
Gamov
Oxtoby
S Ulam
J Von Neuman


E space
A ⊂ E
length
1. m(A) ≥ 0
m(I) = 1
m{p} = 0 p a single point
A, B are disjoint
m(A+B) = m(A) + m(B)

MATHEMATICS

Stan Ulam was first and foremost a mathematician, but his nature was far too expansive to be contained within a single discipline. He saw even the most abstract realms of mathematics in relation to the natural sciences and looked to the natural sciences for inspiration in mathematics. His extraordinary facility for turning a simple question into a bona fide mathematical problem is evident in the mathematics articles presented here. Also evident is his pioneering interest in the use of computers for what he called heuristic studies in mathematics and nonlinear science—what we now call experimental mathematics. Following directly from Stan's example, experimental mathematics has become the primary tool for exploring the complex behavior of nonlinear systems and as such appears as a recurrent theme throughout "The Ulam Legacy."

Stan's great generosity and ingenuity attracted a long list of collaborators—among them the authors who so gladly contributed to this section. Each has a unique experience of the man to relate, and each introduces the reader to one or several areas of mathematics by discussing problems that Stan posed. Our authors—undoubtedly influenced by Ulam—go straight to the heart of the matter and thereby make mathematics a delight to learn.

We start with an article by David Hawkins, a philosopher from the University of Colorado, Boulder, who met Stan at Los Alamos during the Manhattan Project. In his autobiography Stan characterizes Hawkins as "the most talented amateur mathematician I know." During the war he and Hawkins, "teacher" and "student," developed a formalism to describe the multiplication of neutrons during a fission chain reaction. (So began the abstract theory of branching processes later developed with C. J. Everett.) Through Stan's vision the neutron-multiplication problem and other topics discussed by Hawkins (prime numbers, the Monte Carlo method, "sexual" reproduction) took shape as iterative processes of a nonstandard kind, perfectly suited for study on electronic computers. Hawkins, in "The Spirit of Play," has portrayed better than anyone else the fun of working with Stan.

Dan Mauldin, professor of mathematics at North Texas State University and the author of "Probability and Nonlinear Systems," was a major collaborator and close friend of Stan's during the last ten years of his life. Mark Kac, another Polish mathematician from the Lwów school, gave a clue to the source of this strong collaboration: "[Dan] is a first-rate mathematician and he has the Polish soul with regard to mathematics. ... He was on his way to becoming an all-American linebacker on the famous Longhorn team, and he gave it up for mathematics!" Dan's love for mathematics and gift for teaching shine through as he introduces the readers to the basic tools of measure and probability and then shows how to apply these tools to the fashionable and challenging problems in nonlinear science. His tutorial on probability and measure is intended to fill some gaps in our mathematical background as it reminds us of the beauty and precision of the mathematicians' world. Of the three research problems that follow the tutorial, the first and third demonstrate how intuition gained from computer experiments leads to strict mathematical proofs. The second, "Geometry, Invariant Measures, and Dynamical Systems," is most closely tied to the physicists' approach to nonlinear systems. We know now that many deterministic nonlinear systems live on strange attractors, delicate fractal structures that describe a never-repeating orbit confined to a finite region of space. These systems exhibit what appears to be chaotic behavior. How would one describe the long-term behavior of such a system? Dan shows us how to define a probability measure on strange attractors that could be used to calculate the average properties of complex nonlinear systems.

In the article that follows, Paul Stein takes us back a step in history to the computer studies done with Stan in the early sixties on iterations of nonlinear transformations. Their work, apart from leading to perhaps the first discovery of a strange attractor, also led to the 1973 paper by Metropolis, Stein, and Stein on the iteration of the famous one-dimensional logistic map. This paper was a source of inspiration for Feigenbaum's 1976 breakthrough on the universal nature of the transition to chaos by "period doubling." Stein describes the earlier results on period doubling as well as some recent work with Mauldin on an alternate route to chaos that has been observed in chemical experiments.

The last two authors in this section break off from the theme of experimental mathematics and nonlinear systems. Mycielski, a Polish mathematician well known in the field of logic and set theory, took this opportunity to introduce to the non-mathematician two of Ulam's formidable contributions to pure mathematics; first his measurable cardinals, which allow one to talk about orders of infinity well beyond what anyone had dreamed of, and, second, his proof with Oxtoby of the existence of ergodic transformations, one important step toward proving the ergodic hypothesis, the most controversial assumption in the foundations of statistical mechanics. (The significance of ergodicity or the lack thereof is discussed again in the physics section of "The Ulam Legacy").

Ronald Graham from Bell Laboratories closes our foray into mathematics with a little introduction to graph theory. Graham focuses on a problem that illustrates Stan's continuing fascination with quantifying exactly how alike (or different) mathematical objects or structures are. This theme will recur in another form when Walter Goad takes up the question of "alikeness" of DNA sequences in the biology section of "The Ulam Legacy."